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A NOTE ON THE ESTIMATION OF VARIANCE OF INTRA-SIRE REGRESSION HERITABILITY

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SUMMARY

For the variance $4(1-\beta^2)/(N-s-2)$ of intra-sire regression heritability coefficient two estimators can be obtained by replacing β by its BLUE or by its MLE. These two estimators are compared for their relative efficiencies. It is concluded that either of the two could be used with equal advantage.

Keywords: Heritability; Intra-sire regression; Estimators for variance; Mean-squared error; Relative efficiency.

Introduction

Intra-sire regression heritability (h^2) is considered the most reliable measure of additive genetic variability in farm animals. The estimate in this case is obtained by pooling the regression within sire groups in a weighted average. Prabhakaran and Jain [1] have shown that the usual expression for variance of $\hat{h}^2$ is not tenable in view of the random sampling of parents involved and suggested a new expression for the unconditional variance and discussed the procedure for its estimation. The estimator as proposed by these workers is obtained by replacing $\beta = h^2/2$ in the expression for variance by its unbiased estimate b, the estimated intra-sire regression coefficient. The purpose of this paper is to give a new variance eetimator by using the maximum likelihood estimator (MLE) of β in place of β and compare its efficiency relative to the one proposed earlier.

2. The Estimators and their MSE

The expression for unconditional variance as given by Prabhakaran and Jain [1] is

$$V(h_R^2) = 4\,(1 - \beta^2/(N - s - 2) \tag{1}$$

where,

N = the number of paired observations

s = the number of sires considered

$E(h_R^2) = h^2$ and $\beta = \rho = \frac{\sigma_{xy}}{\sigma_x^2}$,

assuming equality in the phenotypic variance among progency (σ_y^2) and dam (σ_x^2)

Let W_{xx}, W_{yy} be the error line (within sires) corrected sum of squares of x and y respectively and W_{xy}, the corresponding sum of products in the analysis of covariance based on the usual random linear model,

$$y_{ij} = s_i + \beta\, x_{ij} + e_{ij}$$

Under the above set-up, the variance estimator of Prabhakaran and Jain [1] is

$$V_1\,(h_R^2) = \frac{4}{(N - s - 2)}\left[1 - (W_{xy}^2/W_{xx})^2\right] \tag{2}$$

where $(W_{xy}/W_{xx}) = b$ is the usual intra-sire regression coefficient. The estimator now proposed is,

$$V_2\,(h_R^2) = \frac{4}{(N - s - 2)}\left[1 - 4\,W_{xy}^2/\,(W_{xx} + W_{yy})^2\right] \tag{3}$$

obtained on replacing β in expression (1) by its MLE,

$$2\,W_{xy}/(W_{xx} + W_{yy})$$

Bias, B_1 due to $V_1\,(h_R^2)$ is seen to be,

$$B_1 = \frac{4}{(N-s-2)}\left[1 - E\left(W_{xy}^2/W_{xx}^2\right)\right] - \frac{4}{(N-s-2)}(1-\beta^2)$$

Since $E\left(W_{xy}^2/W_{xx}^2\right) = \int_{-\infty}^{\infty} b^2 f(b)\, db$,

$$f(b) = \frac{\sigma_x\left[1 + \frac{(b-\beta)^2\sigma_x^2}{\sigma_y^2(1-\rho^2)}\right]^{-\frac{(N-s+1)}{2}}}{\sqrt{\sigma_y^2(1-\rho^2)}\cdot B\left(1/2, \frac{N-s}{2}\right)}$$

$$B_1 = -4(1-\beta^2)/(N-s-2)$$

For obtaining approximate bias B_2 due to $V_2(h_R^2)$ we proceed as follows. Let

$$W_{xy} = E(W_{xy}) + e_1$$

$$W_{xx} = E(W_{xx}) + e_2$$

$$W_{yy} = E(W_{yy}) + e_3$$

Noting that,

$$E(W_{xy}) = (N-s)\beta\sigma^2$$

$$E(W_{xx}) = (N-s)\sigma^2$$

$$E(W_{yy}) = (N-s)\sigma^2,$$

$$E\left[\frac{2W_{xy}}{W_{xx}+W_{yy}}\right]^2 = \beta^2 E\left[\frac{(1+e_1/k)^2}{\left\{1+\frac{(e_2+e_3)\beta}{k}\right\}^2}\right]$$

where $k = (N-s)\beta\sigma^2$

$$= \beta^2 + (\beta^2/k^2)E(e_1^2) - (2\beta^2/k^2)E(e_1 e_2 + e_1 e_3) - (B^4/4k^2)\, E(e_2^2 + e_3^2 + 2e_2e_3)$$

The expectations apprearing in this expression are in fact,

$$E(e_1^2) = V(W_{xy}) = (N-s)(1+\beta^2)\sigma^4$$

$$E(e_2^2) = V(W_{xx}) = 2(N-s)\,\sigma^4$$

$$E(e_3^2) = V(W_{yy}) = 2(N-s)\,\sigma^4$$

$$E(e_1 e_2) = \text{Cov}(W_{xy}, W_{xx}) = 2(N-s)\,\beta\,\sigma^4$$

$$E(e_1 e_3) = \text{Cov}(W_{xy}, W_{yy}) = 2(N-s)\,\beta\,\sigma^4$$

$$E(e_2 e_3) = \text{Cov}(W_{xx}, W_{yy}) = 2(N-s)\,\beta^2\,\sigma^4$$

Accordingly we get

$E[4W_{xy}/(W_{xx}+W_{yy})^2] = \beta^2 + (1 - 8\beta^2 - \beta^4)/(N-s)$ and B_2 is equal to $-4(1-8\beta^2-\beta^4)/(N-s-2)(N-s)$ (4)

Approximate variance of $V_1\,(h_R^2)$ is,

$$V(W_{xy}^2/W_{xx}^2) = \left(\frac{\partial T}{\partial W_{xx}}\right)^2 V(W_{xx}) + \left(\frac{\partial T}{\partial W_{xy}}\right)^2 V(W_{xy})$$

$$+\,2\left(\frac{\partial T}{\partial W_{xx}}\right)\left(\frac{\partial T}{\partial W_{xy}}\right)\text{Cov}(W_{xx}, W_{xy})$$

where

$$T = \frac{W_{xy}^2}{W_{xx}^2}$$

$\frac{\partial T}{\partial W_{xx}}$, the partial derivative of T w. r. t. W_{xx} etc. and W_{xx}, W_{yy} and W_{xy} are to be replaced by their expectations.

This formula on evaluation gives,

$$V[V_1(h_R^2)] = 64\,\beta^2\,(1-\beta^2)/(N-s-2)^2\,(N-s) \qquad (5)$$

Proceeding in a similar fashion we find,

$$V[V_2(h_R^2)] = 64\,\beta^2\,(1-\beta^2)^2/(N-s-2)^2\,(N-s) \qquad (6)$$

The sqared bias being negligible for both B_1 and B_2 we see that the mean-squared errors (MSE) involved are equal to the approximate variances of the estimators given in expressions (5) and (6). It follows immediately that,

$$\text{Efficiency of } V_1 (h_R^2) = V_2 (h_R^2) \,/\, V_1 (h_R^2)$$
$$= (1 - \beta^2) \qquad (7)$$

3. Conclusion

It may be observed that when the population heritability (h^2) is 0.10 ($\beta = 0.05$) the estimator $V_1(h_R^2)$ is as efficient as the MLE based estimator $V_2 (h_R^2)$ and at $h^2 = 0.8$ the efficiency goes down (marginally) to 85%. But such highly heritable characters are seldom encountered in animal breeding work. Hence either of the two estimators could be used with equal advantage.

REFERENCE

[1] Prabhakaran, V. T. and Jain, J. P. (1987) : On the variance of heritability coefficient obtained from intra-sire regression. *The Statistician*, 36 : 25-26.